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Mathematical Modelling of ill-effect of mobile phone on teenagers

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ABSTRACT

In this paper a mathematical model is proposed and analyzed to study the ill-effect of mobile phone on teenagers. By analyzing the model we have found that if inter-specific interference co-efficient of negative influence of mobile phone on teenager and specific growth rate/ production rate co-efficient of mobile phones are not controlled, equilibrium is not exist. Also if Alsoif specific growth rate of manufacturing mobile phone due to the increased use by teenagers, inter-specific interference co-efficient of negative influence of mobile phone on teenagersis too high then it leads to instability. So we along with the government should take proper steps to control the manufacturing rate which gives profit to the company but loss to our society by effecting our future generation and so to the development of our country.

Key words: Autonomous Differential Equations, Equilibrium point, Local Stability, Global Stability.

INTRODUCTION

Mathematical models in ecology discuss about the dynamical behavior of the ecological or environmental system. Biodiversity is the combination of species, genetic and ecological diversity (World Conservation Monitoring Centre (1992)) . Environment is the sum total of all the conditions as well as influence that effect the development and life of the organisms- lowest to the highest including human beings. Along with the diversity, stability and complexity which affect the development of a society were discussed by many researchers (Deka, 2015, Deka, 2015, Dubey *et al.,*1999).

Mobile Phones- the main means through which people gets a platform to various elements of the outside world. These days it has become a necessity in our day to day lives without which daily activities turns lame. We cannot deny the fact that it is useful at times but the negative effects have outdone the positive ones. It has become a serious issue causing many negative effects not only on the children but also on the elderly people. One such is the higher use of social media. It gives rise to cyber bullying- an organization "Enough is enough" conducted a survey that found that due to use of cell phones

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cyber bullying has increased, 95% of teenagers who use social media have witnessed cyber bullying and 33% have been victims themselves. Easy use of social media due to increased use of mobile phones made users unhappier than who used these sites lesser. (Sources: University of Michigan). Mobile phones also have reduced learning and research capabilities; it has become a mere wastage of time for most teenagers. There is lesser motivation in the teenagers and the academic grades too are decreasing for them. So here we have tried to form a model through which we shall try to depict the ill effects of mobile phones on the teenagers.We are conducting a survey by asking few questions (see Annexure) about the use of mobile phone to the students of different departments of Darrang College and find out some information regarding the use and effect of mobile phone on them. Teenagers who are the future of our society are badly affected by the Smart phones largely available nowadays.

MATHEMATICAL MODEL

We consider an ecosystem to model the ill-effect of mobile phone on teenagers. We assume that the dynamics of the system can be governed by the following differential equations:

$$\frac{dT}{dt} = T\left[\left(a_{11} - a_{10}\frac{T}{K}\right) - a_{12}M\right]$$

$$\frac{dM}{dt} = M\left[\left(a_{21} - a_{22}M\right) + a_{23}T\right]$$

$$T(0) \geq 0,\ M(0) \geq 0.$$

Where

T = Density of teenagers.

M =Density of mobile phones.

K = Carrying capacity of teenagers.

a_{11} = Specific growth rate of teenagers.

a_{10} = Intra-specific interference co-efficient of teenagers.

a_{12}=Inter-specific interference co-efficient of negative influence of mobile phone on teenagers.

a_{21} = Specific growth rate/production rate co-efficient of mobile phones.

a_{22} = Intra-specific interference co-efficient of mobile phone.

a_{23}=Specific growth rate of manufacturing mobile phone due to the increased use by teenagers.

$a_{ij}>0;\ i = 1,2^0,\ j = 0, 1,2,3.$

Existence of equlibria

Putting $\frac{dz}{dt} = 0$ (z = T, M) andsolving the equations for T and M we get the following non-negative equilibria, namely-$\mathbf{E_0}(0, 0)$, $E_1(\bar{T}, 0)$, $E_2(0, \bar{\bar{M}})$, $E^*(T^*, M^*)$.

1. The equilibrium $E_0(0, 0)$ is obviously exist.

2.The equilibrium $E_1(\bar{T}, 0)$ is exist as for $\bar{T} \neq 0$, $(a_{11} - a_{10}\frac{\bar{T}}{K}) = 0 \Rightarrow \bar{T} = \frac{a_{11}}{a_{10}} K > 0$;

3.The equilibrium $E_2(0, \bar{\bar{M}})$ is also obviously exist as for $\bar{\bar{M}} \neq 0$, $(a_{21} - a_{22}\bar{\bar{M}}) = 0 \Rightarrow \bar{\bar{M}} = \frac{a_{21}}{a_{22}} > 0$;

4. Existence of the equilibrium $E^*(T^*, M^*)$:

Since $T^*, M^* \neq 0$,

$\therefore (a_{11} - a_{10}\frac{T^*}{K}) - a_{12}M^* = 0$ and $a_{21} - a_{22}M^* + a_{23}T^* = 0$

$\Rightarrow \frac{a10}{K}T^* + a_{12}M^* - a_{11} = 0$ ....(2.1) $\Rightarrow a_{23}T^* - a_{22}M^* + a_{21} = 0$ ....(2.2)

Solving the equations *(2.1), (2.2)*; we get

$$\Rightarrow \quad T^* = \frac{(a_{11}a_{22} - a_{21}a_{12})}{(a_{23}a_{12} + \frac{a_{22}a_{10}}{K})}, \qquad M^* = \frac{\left(\frac{a_{21}a_{10}}{K} + a_{23}a_{11}\right)}{(a_{23}a_{12} + \frac{a_{22}a_{10}}{K})} > 0 \text{ (obvious).}$$

But T^* is positive, if $a_{11}a_{22} > a_{21}a_{12}$ .... (2.3)

Also, we have

$$\frac{dT}{dM} = -a_{12} < 0, \quad \frac{dT}{dM} = a_{23} > 0$$

Hence two isoclines intersect at a unique point E^*.

E^* exists only if the condition (2.3) is maintained. Thus if a_{12}-inter-specific interference co-efficient of negative influence of mobile phone on teenager and a_{21} - specific growth rate/production rate co-efficient of mobile phones are not controlled, equilibrium of E^* is not exist.

MATERIALS AND METHODS

Stability analysis of the equilibria

The local stability of the equilibria can be studied by computing variational matrices corresponding to each equilibria by using Gershgorion's Theorem and Routh–Hurwitz criteria (La Salle and La Salle J. and Lefschetz S., 1961, Freedman, 1978, Rao, 1981).

RESULTS AND DISCUSSION

Variational matrices:

$$J(T, M) = \begin{bmatrix} a_{11} - 2\frac{a_{10}}{K}T - a_{12}M & -a_{12}T \\ a_{23}M & a_{21} - 2a_{22}M + a_{23}T \end{bmatrix}$$

$$J^*(T^*, M^*) = \begin{bmatrix} -\frac{a_{10}}{K}T^* & -a_{12}T^* \\ a_{23}M^* & -a_{22}M^* \end{bmatrix}$$

Characteristic equation:

$$|J - \lambda I| = \begin{vmatrix} a_{11} - 2\frac{a_{10}}{K}T - a_{12}M - \lambda & -a_{12}T \\ a_{23}M & a_{21} - 2a_{22}M + a_{23}T - \lambda \end{vmatrix} = 0$$

i. Characteristic equation at E_0 (0, 0),

From these matrices we note the following results.

$$\begin{vmatrix} a_{11} - \lambda & 0 \\ 0 & a_{21} - \lambda \end{vmatrix} = 0$$

$$\Rightarrow (a_{11} - \lambda)(a_{21} - \lambda) = 0$$
$$\therefore \lambda_1 = a_{11} > 0, \quad \lambda_2 = a_{21} > 0$$

Equilibrium E_0 (0, 0) is unstable by Gershgorion's Theorem

ii. Characteristic equation at E_1 ($\overline{T}$, 0)

$$\begin{vmatrix} -\frac{a_{10}}{K}\overline{T} - \lambda & -a_{12}\overline{T} \\ 0 & a_{21} + a_{23}\overline{T} - \lambda \end{vmatrix} = 0$$

$\lambda_1 = -\frac{a_{10}}{K}\overline{T} < 0, \lambda_2 = a_{21} + a_{23}\overline{T} > 0$ as $\overline{T} > 0$.

E_1 ($\overline{T}$, 0) is a saddle point, so it is unstable.

iii. Characteristic equationat E_2 (0, $\overline{M}$),

$$\begin{vmatrix} a_{11} - a_{12}\overline{M} - \lambda & 0 \\ a_{23}\overline{M} & -a_{22}\overline{M} - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda_1 = a_{11} - a_{12}\bar{\bar{M}} < 0, \text{if}\ \bar{\bar{M}} = \frac{a_{21}}{a_{22}} > \frac{a_{11}}{a_{12}} \text{ i.e. } a_{11}a_{22} < a_{21}a_{12},\ \text{but}\ a_{11}a_{22} > a_{21}a_{12}\ \text{by (2.3)}$$

Also $\lambda_2 = -\,a_{22}\bar{\bar{M}} < 0$ as $\bar{\bar{M}} = \frac{a_{21}}{a_{22}} > 0$ .So $E_2(0, \bar{\bar{M}})$ is stable if $\bar{\bar{M}} > \frac{a_{11}}{a_{12}}$ holds, and is a saddle point if (2.3) holds.

iv. Characteristic equation at E^* (T^*, M^*)

$$|J^* - \lambda I| = \begin{vmatrix} -\frac{a_{10}}{K}T^* - \lambda & -a_{12}T^* \\ a_{23}M^* & -a_{22}M^* - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \left(-\frac{a_{10}}{K}T^* - \lambda\right)(-a_{22}M^* - \lambda) + a_{12}a_{23}T^*M^* = 0$$

$$\Rightarrow \lambda^2 + \left(\frac{a_{10}}{K}T^* + a_{22}M^*\right)\lambda + \left(\frac{a_{10}}{K}a_{22} + a_{12}a_{23}\right)T^*M^* = 0$$

$\lambda_1 = (\frac{a_{10}}{K}T^* + a_{22}M^* > 0$ if $T^* > 0$,

$\lambda_2 = (\frac{a_{10}}{K}a_{22} + a_{12}a_{23})T^*M^* > 0$, (by Routh-Hurwitz Criteria)

Both the conditions are satisfied if $T^* > 0$, that is, if the condition (2.3) is satisfied.

Global stability:

In order to investigate the global stability behavior of the interior equilibrium E^*, we first establish the following lemma which establishes a region of attraction for the system (1),

LEMMA: The set $\Omega = \{(T, M): 0 \le T \le T_m;\ 0 \le M \le M_m\}$ attracts all the solutions initially in the positiveorthant.

Where,

$T_m = \frac{a_{11}K}{a_{10}}$, $M_m = \frac{a_{10}a_{21} + a_{11}a_{23}K}{a_{10}a_{22}}$

THEOREM: If the following in equality holds, the equilibrium E^* is globally asymptotically stable with respect to all solutions initially is the positive orthant. (For proof see Appendix)

$$4a_{12}a_{23} < \frac{a_{10}a_{22}}{K}$$

Then the system is globally asymptotically stable.

Survey report:

ACADEMICS	52
ENTERTAINMENT	68
TOTAL STUDENTS	120

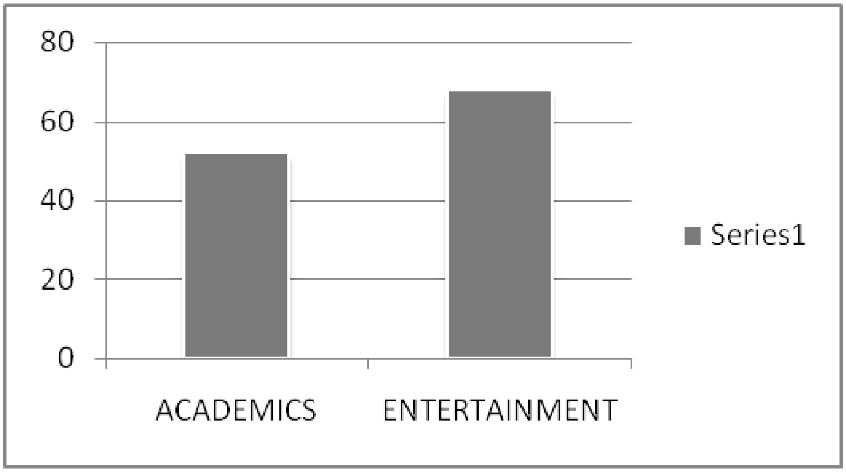

Figure 1. Reflects entertainment is the major priority.

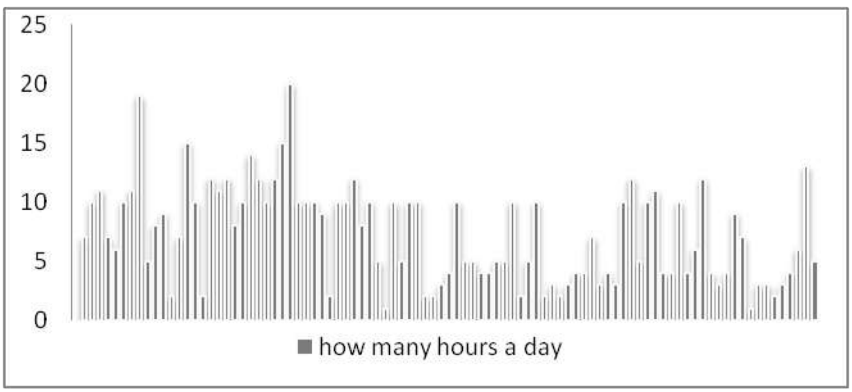

Figure 2. Use mobile phone at average 10 hours a day. Some students use it maximum of 15 hours per day.

NO OF STUDENTS ADDICTED	59
NO OF STUDENTS NOT ADDICTED	33
SUM	92

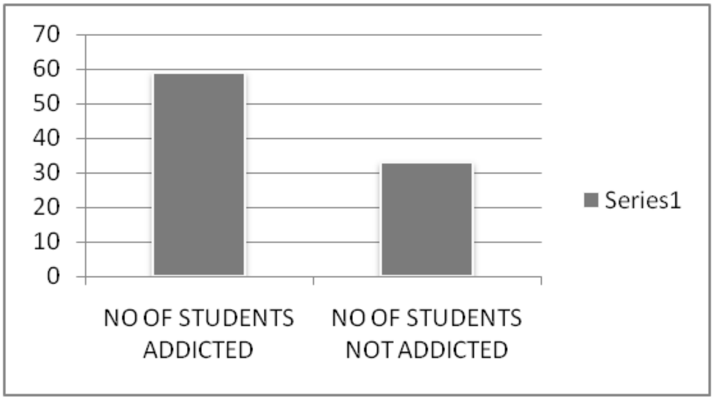

Figure 3. More number of students are addicted.

By analyzing the model we have found that if a_{12}-inter-specific interference co-efficient of negative influence of mobile phone on teenager and a_{21}- specific growth rate/production rate co-efficient of mobile phones are not controlled, equilibrium of E* is not exist. Also if a_{23}-specific growth rate of manufacturing mobile phone due to the increased use by teenagers, a_{12}-Inter-specific interference co-efficient of negative influence of mobile phone on teenagersis too high then it leads to instability. So we along with the government should take proper steps to control the manufacturing rate which gives profit to the company but loss to our society by effecting our future generation and so to the development of our country.

APPENDIX

Proof of the Lemma: We have

$$\frac{dT}{dt} = T\left[\left(a_{11} - \frac{a_{10}}{K}T\right) - a_{12}M\right]$$

$$\Rightarrow \frac{dT}{dt} \leq T\left(a_{11} - \frac{a_{10}}{K}T\right) \Rightarrow \frac{dT}{T\left(a_{11} - \frac{a_{10}}{K}T\right)} \leq dt$$

$$\Rightarrow\left(\frac{\frac{1}{a_{11}}}{T}+\frac{a_{10}/a_{11}K}{\left(a_{11}-\frac{a_{10}}{K}T\right)}\right)dT \leq dt$$

$$\Rightarrow\frac{1}{a_{11}}\log T-\frac{K}{a_{10}}\frac{a_{10}}{Ka_{11}}\frac{d(a_{11}-\frac{a_{10}}{K}T)}{(a_{11}-\frac{a_{10}}{K}T)} \leq dt$$

$$\Rightarrow\frac{1}{a_{11}}\left[\log T-\log(a_{11}-\frac{a_{10}}{K}T)\right] \leq dt$$

$$\Rightarrow\log\frac{T}{a_{11}-\frac{a_{10}}{K}T} \leq a_{11}dt$$

$$\Rightarrow\frac{T}{a_{11}-\frac{a_{10}}{K}T} \leq e^{a_{11}t}$$

$$\Rightarrow T \leq (a_{11}-\frac{a_{10}}{K}T)e^{a_{11}t}$$

$$\Rightarrow T(1+\frac{a_{10}}{K}e^{a_{11}t}) \leq a_{11}e^{a_{11}t}$$

$$\Rightarrow T \leq \frac{a_{11}e^{a_{11}t}}{1+\frac{a_{10}}{K}e^{a_{11}t}}=\frac{a_{11}e^{a_{11}t}}{e^{a_{11}t(e^{-a_{11}t}+\frac{a_{10}}{K})}}$$

$$\Rightarrow T \leq \frac{a_{11}}{e^{-a_{11}t}+\frac{a_{10}}{K}}$$

Therefore, when $t\to\infty$, $T \leq \frac{a_{11}}{\frac{a_{10}}{K}}=\frac{a_{11}K}{a_{10}}=T_m$

$$\Rightarrow\lim_{t\to\infty}, Sup\, T(t)=T_m$$

Also

$$\frac{dM}{dt}=M[a_{21}-a_{22}M+a_{23}T] \leq M\left[\left(a_{21}+a_{23}\frac{a_{11}K}{a_{10}}\right)-a_{22}M\right]$$

$$\Rightarrow\frac{dM}{dt} \leq \left(a_{21}+\frac{a_{11}a_{23}K}{a_{10}}\right)M\left[1-\frac{a_{22}M}{a_{21}+\frac{a_{11}a_{23}K}{a_{10}}}\right]$$

$$\Rightarrow\frac{dM}{M\,(1-a_{22}M/P)} \leq \left(a_{21}+\frac{a_{11}a_{23}K}{a_{10}}\right)dt=Pdt$$, where $P=a_{21}+\frac{a_{11}a_{23}K}{a_{10}}$.

$$\Rightarrow\left(\frac{1}{M}+\frac{a_{22}/P}{1-a_{22}\,M/P}\right)dM \leq Pdt$$

$$\Rightarrow\log M-\log(1-\frac{a_{22}\,M}{P}) \leq Pt$$

$$\Rightarrow\log\frac{M}{1-a_{22}\,M/P} \leq Pt$$

$\Rightarrow \frac{M}{1-a_{22}\,M/P} \leq e^{Pt}$

$\Rightarrow M \leq (1-\frac{a_{22}\,M}{P})e^{Pt}$

$\Rightarrow M \leq \frac{1}{e^{-Pt}+\frac{a_{22}}{P}}$

Therefore, when $t \to \infty$, $M \leq M_m = \frac{P}{a_{22}} = \frac{a_{10}a_{21}+a_{11}a_{23}K}{a_{10}a_{22}}$.

Proof of the Theorem: Let us consider the following positive definite Liaponuvfunction aroundE^*

$$V(T,M) = \left[T - T^* - T^* ln\frac{T}{T^*}\right] + C\left[M - M^* - M^* ln\frac{M}{M^*}\right]$$

$$\Rightarrow \dot{V}(T,M) = \left[\frac{dT}{dt} - 0 - \frac{T^*}{\frac{T}{T^*}}\frac{1}{T^*}\frac{dT}{dt}\right] + C\left[\frac{dM}{dt} - \frac{M^*}{\frac{M}{M^*}}\frac{1}{M^*}\frac{dM}{dt}\right]$$

$$= \left[1 - \frac{T^*}{T}\right]\frac{dT}{dt} + c\left[1 - \frac{M^*}{M}\right]\frac{dM}{dt}$$

$$\Rightarrow \dot{V}(T,M) = \frac{1}{T}[T - T^*]T\left[a_{11} - \frac{a_{10}}{K}T - a_{12}M\right] + \frac{C}{M}[M - M^*]M[a_{21} - a_{22}M + a_{23}T]$$

$$=(T - T^*)\left(a_{11} - \frac{a_{10}}{K}T - a_{12}M - a_{11} + \frac{a_{10}}{K}T^* + a_{12}M^*\right) + C(M - M^*)(a_{21} - a_{22}M + a_{23}T - a_{21} + a_{22}M^* - a_{23}T^*)$$

$$=(T - T^*)\left\{-\frac{a_{10}}{K}(T - T^*) - a_{12}(M - M^*)\right\} + C(M - M^*)\{-a_{22}(M - M^*) + a_{23}(T - T^*)\}$$

$$=\frac{-a_{10}}{K}(T - T^*)^2 - a_{12}(T - T^*)(M - M^*) - Ca_{22}(M - M^*)^2 + Ca_{23}(M - M^*)(T - T^*)$$

$$= -\frac{a_{10}}{K}(T - T^*)^2 + (a_{23}C - a_{12})(T - T^*)(M - M^*) - Ca_{22}(M - M^*)^2$$

$$=- A_{11}(T - T^*)^2 + A_{12}(T - T^*)(M - M^*) - A_{22}(M - M^*)^2 .$$

Where $A_{11} = \frac{a_{10}}{K}$, $A_{12} = (a_{23}C - a_{12})$, $A_{22} = Ca_{22}$

Let us choose $C = \frac{a_{12}}{a_{23}}$

Then $\dot{V}$ is a negative definite if $A_{12}^2 < A_{11}A_{22}$, i. e., if $4a_{12}a_{23} < \frac{a_{10}a_{22}}{K}$

Then the system is globally asymptotically stable.

REFERENCES

Deka, Barnali Das 2015 Mathematical modeling on the ill effect of PartheniumHysterophorus Linn inhuman health-A case study of Tezpur-Subdivision in Sonitpur District, Assam. "*Proceeding of ICFM 2015*", International Conference on frontiers in Mathematics 2015, Gauhati University, dpp 35-38.

Deka, Barnali Das 2015 Modeling the interaction of Susceptible prey with predator in infected environment, *Mathematical Forum*, vol. XXI, : 51-61.

Dubey B. and Das B 1999, Models for the survival of species dependent on resource in industrial environment, *Journal of Mathematical analysis and Application*, 231 : 374 – 396.

Freedman H. I., 1978 Deterministic Mathematical Model in Population Ecology", HIFR Consulting, Edmonton..

La Salle J. and Lefschetz S 1961 Stability by Liaponuv's Direct Method and Application. Academic Press, New York.

RaoM. R. M. 1981 Ordinary Differential Equations-Theory and Applications, Arnold, London,.