Northeast Journal of Contemporary Research, December 2020. Vol. 7 No. 1, pp 32-48
ISSN 2349-3089

A new PBE2 distribution and life expectancy prediction for India

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ABSTRACT

The Binomial Exponential 2 (BE2) Distribution was proposed by Bakoch et al. as a distribution of a random sum of independent exponential random variables, when sample size has zero truncated binomial distribution. In this article we analyze power binomial exponential 2(PBE2) Distribution, Proposed by Habibi and Asgharzadeh (2017) which is a generalization of BE2 Distribution which offers more flexible model for studying life time data with monotone as well as non-monotone failure rates than the BE2 distribution. From the analysis we have observed that the present PBE2 distribution is not suitable for fitting the life expectancy of Indian data. Therefore, a new transformation is necessary to fit life expectancy of Indian data. Here we have developed a New Power Binomial Exponential 2 (NPBE2) distribution which is suitable for fitting the life expectancy of Indian data.

Key words: Binomial Exponential 2 distribution; Power Binomial Exponential 2 distribution; Survival function; Life expectancy

INTRODUCTION

To model life time data and phenomena related to life time many probability distributions have been used. Of them Gamma and Weibull distributions have been extensively used in the literature for monotone failure rates. The limitation of the Gamma distribution is that its distribution function or the survival function cannot be expressed in a closed form. The Weibull distribution is comparatively popular than Gamma distribution because it has closed form survival and hazard rate functions (Habibi and Asgharzadeh, 2017). However, reliability and survival studies commonly encounter non-monotone failure rates such as the bathtub shaped and unimodal failure rates. Unfortunately, Weibull and Gamma distributions do not provide a reasonable parametric fit for such rates. Further, there are situations where mortality reaches a peak after some finite period and then slowly declines (Gupta and Gupta, 1998).

Several generalizations of Gamma and Weibull distributions have been introduced in the literature for modeling non-monotone failure rate data (Stacy, 1962; Mudholkar and Srivastava, 1993).

Bakouch et al. (2014) introduced the Binomial-Exponential 2 (BE2) distribution as an alternative to the Weibull and Gamma distributions. The probability density function (pdf) of BE2 distribution is given by-

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$$f(t) = \left\{1 + \frac{(\lambda t - 1)\theta}{2-\theta}\right\} \lambda e^{-\lambda t} \quad ; t > 0;\ \lambda > 0;\ 0 \le \theta \le 1 \qquad (1)$$

Where θ is the shape parameter and λ is the scale parameter.
The cumulative distribution function (cdf) is given by

$$F(t) = 1 - \left\{1 + \frac{\lambda\theta t}{2-\theta}\right\} e^{-\lambda t} \qquad ; t > 0$$

Now, Survival function of t is given by

$$S(t) = \left\{1 + \frac{\lambda\theta t}{2-\theta}\right\} e^{-\lambda t} \qquad ; t > 0$$

The pdf (1) can be written as

$$f(t) = p\lambda e^{-\lambda t} + (1-p)\lambda^2 t e^{-\lambda t} \qquad ; t > 0$$

where, $p = \frac{2(1-\theta)}{2-\theta}$.

So, the BE2 distribution is a mixture of Exponential distribution (with scale parameter λ), and a Gamma distribution (with shape parameter 2 and scale parameter λ), with mixing proportion p.
Note that the exponential distribution arises when $\theta = 0$, and for $\theta = 1$ the BE2 distribution reduces to the Gamma distribution with shape parameter 2 and scale parameter λ.
The BE2 distribution allows for increasing rates only. So, this distribution is not suitable for modeling non-monotone failure rate data. Non-monotone failure rates are very important in practice, particularly modeling life expectancies at different ages in life tables of developing countries. In most of the developing countries high rates of infant and childhood mortality result in lower values of life expectancy at birth than at other ages (Romo and Becker, 2011). In India, the Sample Registration System (SRS) life tables show that the highest life expectancy occurred at age 5 till 1980 and then crossed over to age 1 during 1981-85 and remained at that age till date. In all the major states of India, except Kerala, the situation is same. In Kerala, whose demographic features are like the developed countries, the highest life expectancy occurred at birth right from the time since when SRS provides life tables, i.e., 1970-75 (Sarma and Choudhury, 2014).

Habibi and Asgharzadeh (2017) proposed a more flexible distribution for studying non monotone failures. They proposed a new three-parameter generalization of the BE2 distribution. Due to the great flexibility of the failure rate function of the new model, it is suitable for modeling many sets of lifetime data with monotonic as well as non-monotonic failure rates. Further, it has closed form expressions for survival and hazard rate functions. This new extension of BE2 distribution is obtained by considering the power transformation $X = T^{\frac{1}{\alpha}}$, where T be a BE2 random variable with p.d.f. (1). The p.d.f. of X is then,

$$f(x) = \alpha\lambda x^{\alpha-1}\left[1 + \frac{(\lambda x^{\alpha} - 1)\theta}{2-\theta}\right] e^{-\lambda x^{\alpha}} \qquad ; x, \alpha, \lambda > 0 \text{ and } 0 \le \theta \le 1$$

And the survival function of X is

$$S(x) = P(X > x) = \left(1 + \frac{\lambda\theta x^{\alpha}}{2-\theta}\right) e^{-\lambda x^{\alpha}} \qquad ; x, \alpha, \lambda > 0 \text{ and } 0 \le \theta \le 1$$

They named this distribution as Power-Binomial-Exponential 2 (PBE2) distribution.

Habibi and Asgharzadeh (2017) also gave the pdf for Residual (or remaining) life random variable under PBE2 distribution.

Given survival to time $t \geq 0$, the residual life is the period from time t until the time of failure. More specifically, if X is the life of a component, then the conditional random variable $X_{(t)} = X - t|X > t$ is called the residual life of the random variable.

The survival function of the residual life $X_{(t)}$, $t \geq 0$ for the new distribution is

$$S_{X_{(t)}}(x) = \frac{S(x+t)}{S(t)} \qquad ; x > 0$$

And the corresponding pdf is

$$F_{X_{(t)}}(x) = \frac{f(x+t)}{S(t)}$$

$$= \frac{\alpha\lambda(x+t)^{\alpha-1}[2-2\theta+\lambda\theta(x+t)^{\alpha}]}{[2-\theta+\lambda\theta(x+t)^{\alpha}]}$$

The mean residual life (popularly known as life expectancy in Demography) is given by

$$K(t) = E\left[X_{(t)}\right] = \frac{1}{S(t)} \int_{t}^{\infty} x f(x) dx - t$$

$$= \frac{(2-\theta)e^{\lambda t^{\alpha}}}{2-\theta+\lambda\theta t^{\alpha}} L(1,t) - t \qquad ; t \geq 0$$

Where,

$$L(1,t) = \lambda^{-\frac{1}{\alpha}}\left[\left(1 - \frac{\theta}{2-\theta}\right)\Gamma\left(\frac{1}{\alpha} + 1, \lambda x^{\alpha}\right) + \left(\frac{\theta}{2-\theta}\right)\Gamma\left(\frac{1}{\alpha} + 2, \lambda x^{\alpha}\right)\right]$$

We have tried different combinations of the parameters to fit the life expectancy curve of India from SRS based life tables. We have found no single value of the parameters that satisfy the life expectancy curve. Finally, for $\lambda = 1$ and $\theta = \frac{1}{2}$ and varying values of α, life expectancy at different ages can be obtained. That is, $\alpha = \alpha(t)$. However, we have observed that for the present PBE2 distribution, the values of α becomes nearly zero at old ages. Therefore, a new transformation is necessary to fit the life expectancy of Indian data.

Objectives:

The following are the objectives of the present study:

1. To develop a new three-parameter generalization of the BE2 distribution suitable for studying monotone as well as non-monotone failure rates.
2. To determine the parameters $\alpha = \alpha(t)$ for different ages (t) from life expectancies in six SRS life tables of India across age and across time periods.
3. To predict the life expectancies for a future time period by fixing two of the three parameters at constant levels and studying the trend of the third.

MATERIALS AND METHODS

We have taken six SRS life tables of India (Male, Total) from 2008-12 to 2013-17 as our data. A New three parameter distribution is obtained by the transformation $X = T^{\alpha}$ (instead of the transformation $X = T^{\frac{1}{\alpha}}$ as taken by Habibi and Asgharzadeh (2017) in the BE2 distribution.

The probability differential of BE2 distribution is given by

$$dF(t) = \left\{1 + \frac{(\lambda t-1)\theta}{2-\theta}\right\}\lambda e^{-\lambda t}dt \qquad ; t > 0$$

By considering the transformation $X = T^{\alpha}$, The probability differential of X is obtained as

$$dF(x) = \frac{\lambda}{\alpha}x^{\frac{1}{\alpha}-1}\left\{1 + \frac{\left(\lambda x^{\frac{1}{\alpha}}-1\right)\theta}{2-\theta}\right\}e^{-\lambda x^{\frac{1}{\alpha}}}dx$$

Therefore, the pdf of X is given by

$$f(x) = \frac{\lambda}{\alpha}x^{\frac{1}{\alpha}-1}\left\{1 + \frac{\left(\lambda x^{\frac{1}{\alpha}}-1\right)\theta}{2-\theta}\right\}e^{-\lambda x^{\frac{1}{\alpha}}} \qquad ; x > 0 \qquad (2)$$

If we put $\lambda=1$,$\theta = \frac{1}{2}$ in (1) we have pdf of X as

$$f(x) = \frac{1}{\alpha}x^{\frac{1}{\alpha}-1}\left\{1 + \frac{\left(x^{\frac{1}{\alpha}}-1\right)}{3}\right\}e^{-x^{\frac{1}{\alpha}}} \qquad ; x > 0 \qquad (3)$$

and survival function of X is given by

$$S(x) = P(X > x) = \left\{1 + \frac{x^{\frac{1}{\alpha}}}{3}\right\} e^{-x^{\frac{1}{\alpha}}} \qquad ; x > 0, \lambda > 0 \qquad (4)$$

The survival function of the residual life $X_{(t)} = X - t|X > t$, for $t \geq 0$ for the new distribution is

$$S_{X_{(t)}}(x) = \frac{S(x+t)}{S(t)} \qquad ; x > 0$$

And the corresponding pdf is

$$f_{X_{(t)}}(x) = \frac{f(x+t)}{S(t)}$$

$$= \frac{\frac{1}{\alpha}(x+t)^{\frac{1}{\alpha}-1}\left\{1+\frac{(x+t)^{\frac{1}{\alpha}-1}}{3}\right\}e^{-(x+t)^{\frac{1}{\alpha}}}}{S(t)}$$

Where S(.) is the survival function given by (3) , consequently the hazared rate function of $X_{(t)}$ is

$$h_{X_{(t)}}(x) = \frac{f_{X_{(t)}}(x)}{S_{X_{(t)}}(x)}$$

$$= \frac{f(x+t)}{S(x+t)}$$

$$= \frac{\frac{1}{\alpha}(x+t)^{\frac{1}{\alpha}-1}\{2+(x+t)^{\frac{1}{\alpha}}\}}{\{3+(x+t)^{\frac{1}{\alpha}}\}}$$

Further, the mean residual life (life expectancy) is obtain as follows

$$K(t) = E(X_{(t)})$$

$$= \frac{1}{S(t)}\int_t^{\infty} x f(x) dx - t$$

$$= \frac{1}{S(t)}\int_t^{\infty} x \frac{1}{\alpha} x^{\frac{1}{\alpha}-1}\left\{1 + \frac{x^{\frac{1}{\alpha}-1}}{3}\right\} e^{-x^{\frac{1}{\alpha}}} dx - t$$

$$= \frac{1}{S(t)}\left\{\frac{2}{3}\Gamma(\alpha+1,t) + \frac{1}{3}\Gamma(\alpha+2,t)\right\} - t$$

$$= \frac{3}{(3+t^{\frac{1}{\alpha}})e^{-t^{\frac{1}{\alpha}}}}\left\{\frac{2}{3}\Gamma(\alpha+1,t) + \frac{1}{3}\Gamma(\alpha+2,t)\right\} - t$$

Or $$K(t) = \frac{e^{t^{\frac{1}{\alpha}}}}{3+t^{\frac{1}{\alpha}}}\{2\Gamma(\alpha+1,t)+\Gamma(\alpha+2,t)\} - t \quad (5)$$

Note that in this case α varies with t, i.e., $\alpha = \alpha(t)$.

RESULTS AND DISCUSSION

We have estimated different values of α to obtain the life expectancies at different ages from the SRS life tables (Table 1). From the variations of α over age and over time period we have found that α varies linearly over time period and over age. We have fitted a set of regression lines (Table 2) to fit α for different ages (t) over years, where years are the mid-year of the time period of SRS life tables (for example year =2010 for the SRS life table of 2008-12). Tables 3.1 to 3.6 give the estimated α's for different ages and the observed and estimated life expectancies for the SRS life tables we considered. Putting Year = 2016 we can predict the α's for different ages for the 2014-18 period. Then we can use these values to estimate K(t) for different t by (5). That is, we can estimate the life expectancies for different ages for the period 2014-18 (which have not been published yet) and consequently, the whole life table for that period by indirect methods. Table 4 presents the predicted life expectancies for the period 2014-18.Similarly we can estimate the life expectancies for different ages for the periods 2015-19;2016-20;2017-21;2018-22….etc. Table 5,6,7 and 8 presents the predicted life expectancies for the period 2015-19;2016-20;2017-21 and 2018-22 respectively.

Table 1. Estimated values of α across time period and across age

YEARS	α					
	2008-12	**2009-13**	**2010-14**	**2011-15**	**2012-16**	**20013-17**
0	4.094	4.098	4.103	4.108	4.112	4.116
1	3.687	3.690	3.693	3.697	3.701	3.704
5	3.8319	3.8343	3.8371	3.8396	3.8416	3.8441
10	4.7786	4.7805	4.7818	4.7841	4.7861	4.7876
15	5.8424	5.8442	5.8452	5.8472	5.8489	5.8498
20	6.9018	6.9033	6.9043	6.9055	6.9069	6.9084
25	7.9395	7.9404	7.9416	7.9426	7.9438	7.9448
30	8.9539	8.9547	8.9555	8.9563	8.9576	8.9584
35	9.9466	9.9474	9.9481	9.9489	9.9499	9.9507
40	10.9209	10.9215	10.9223	10.9231	10.9237	10.9242
45	11.8783	11.8791	11.8794	11.8801	11.8807	11.8811
50	12.8213	12.8216	12.8223	12.8226	12.8233	12.8236
55	13.7508	13.7514	13.7518	13.7521	13.7527	13.7531
60	14.6692	14.6698	14.6701	14.6704	14.6711	14.6714

65	15.5771	15.5774	15.5777	15.5780	15.5786	15.5786
70	16.4763	16.4763	16.4763	16.4763	16.4769	16.4769
75	17.3672	17.3668	17.3662	17.3660	17.3660	17.3660
80	18.2517	18.2496	18.2480	18.2475	18.2472	18.2470
85	19.1248	19.1244	19.1228	19.1221	19.1218	19.1213

Table 2. The parameters of fitted regression lines for α at age t over different years [α (t, year) = a(t)+b(t)*year]:

Age (t)	a(t) (Intercept)	b(t) (Slope)	R^2
0	-4.92233	0.004486	0.999
1	-3.31967	0.003486	0.997
5	-1.0724	0.00244	0.999
10	1.097367	0.001831	0.998
15	2.793033	0.001517	0.995
20	4.317533	0.001286	0.996
25	5.774367	0.001077	0.999
30	7.116067	0.000917	0.993
35	8.2926	0.000823	0.997
40	9.548367	0.000683	0.994
45	10.75853	0.000557	0.990
50	11.8507	0.000483	0.987
55	12.84923	0.000449	0.992
60	13.79633	0.000434	0.984
65	14.9224	0.000326	0.967
70	16.2005	0.000137	0.686
75	17.86087	-0.00025	0.804
80	20.0425	-0.00089	0.836
85	20.61787	-0.00074	0.940

Table 3.1: Estimated α, Γ(α+1, *t*), Γ(α+2, *t*), estimated and SRS life expectancies for different ages, India (male, Total, 2008-12):

2008-12					
Age (t)	**α**	$\Gamma(\alpha+1, t)$	$\Gamma(\alpha+2, t)$	**Est. L.E**	**SRS L.E**
0	4.094	27.67695	140.9864	65.45	65.4
1	3.687	15.05138	70.91368	67.65	67.6
5	3.8319	7.66363	53.0949	64.32	64.3
10	4.7786	4.685209	54.34207	59.63	59.6
15	5.8424	3.534462	58.29325	54.81	54.8
20	6.9018	2.904654	62.2698	50.14	50.1
25	7.9395	2.496832	65.92406	45.63	45.6
30	8.9539	2.204815	69.18356	41.17	41.1
35	9.9466	1.981212	72.03625	36.67	36.6
40	10.9209	1.809324	74.80627	32.38	32.3
45	11.8783	1.670556	77.40146	28.15	28.1
50	12.8213	1.560284	80.07784	24.16	24.1
55	13.7508	1.468378	82.68999	20.25	20.2
60	14.6692	1.397078	85.64945	16.75	16.7
65	15.5771	1.340283	88.84839	13.54	13.5
70	16.4763	1.297951	92.75328	11.00	10.9
75	17.3672	1.2726	97.0876	8.87	8.8
80	18.2517	1.2634	102.694	7.84	7.1
85	19.1248	1.2404	107.025	5.73	5.7

Table 3.2. Estimated α, Γ(α+1, *t*), Γ(α+2, *t*), estimated and SRS life expectancies for different ages, India (male, Total, 2009-13):

2009-13					
Age (t)	**α**	**Γ(α+1, *t*)**	**Γ(α+2, *t*)**	**Est. L.E**	**SRS L.E**
0	4.098	27.84654	141.9616	65.88	65.8
1	3.69	15.11678	71.26556	67.98	67.9
5	3.8343	7.698836	53.34566	64.62	64.6
10	4.7805	4.707009	54.59654	59.94	59.9
15	5.8442	3.552317	58.58849	55.15	55.1
20	6.9033	2.918031	62.55695	50.45	50.4
25	7.9404	2.504196	66.11865	45.83	45.8
30	8.9547	2.2109	69.37461	41.36	41.3
35	9.9474	1.986915	72.24368	36.87	36.8
40	10.9215	1.813368	74.97353	32.54	32.5
45	11.8791	1.67569	77.63937	28.37	28.3
50	12.8216	1.562128	80.17251	24.25	24.2
55	13.7514	1.471933	82.89027	20.43	20.4
60	14.6698	1.400538	85.86187	16.94	16.9
65	15.5774	1.342115	88.95959	13.64	13.6
70	16.4763	1.297951	92.75328	11.00	10.9
75	17.3668	1.2518	96.9184	8.69	8.7
80	18.2496	1.2634	101.75	7.05	7.0
85	19.1244	1.2382	106.834	5.57	5.5

Table 3.3. Estimated α, Γ(α+1, *t*), Γ(α+2, *t*), estimated and SRS life expectancies for different ages, India (male, Total, 2010-14):

2010-14					
Age (t)	**α**	**Γ(α+1, *t*)**	**Γ(α+2, *t*)**	**Est. L.E**	**SRS L.E**
0	4.103	28.06011	143.1908	66.44	66.4
1	3.693	15.18249	71.61931	68.31	68.3
5	3.8371	7.740116	53.63973	64.98	64.9
10	4.7818	4.721984	54.77135	60.16	60.1
15	5.8452	3.562275	58.75315	55.34	55.3
20	6.9043	2.926983	62.74912	50.66	50.6
25	7.9416	2.514048	66.37899	46.10	46.1

30	8.9555	2.217002	69.56618	41.55	41.5
35	9.9481	1.991918	72.42566	37.05	37.0
40	10.9223	1.818775	75.19713	32.76	32.7
45	11.8794	1.677619	77.72878	28.46	28.4
50	12.8223	1.56644	80.39384	24.45	24.4
55	13.7518	1.474307	83.02406	20.55	20.5
60	14.6701	1.402271	85.96779	17.04	17.0
65	15.5777	1.343234	89.07091	13.74	13.7
70	16.4763	1.297951	92.75328	11.00	10.9
75	17.3662	1.266	96.583	8.43	8.5
80	18.248	1.2403	100.815	6.23	6.4
85	19.1228	1.2256	105.745	4.64	4.9

Table 3.4. Estimated α, Γ(α+1, *t*), Γ(α+2, *t*), estimated and SRS life expectancies for different ages, India (male, Total, 2011-15):

2011-15					
Age (t)	**α**	**Γ(α+1, *t*)**	**Γ(α+2, *t*)**	**Est. L.E**	**SRS L.E**
0	4.108	28.27548	144.4312	66.99	66.9
1	3.697	15.2706	72.0939	68.75	68.7
5	3.8396	7.777163	53.90369	65.30	65.2
10	4.7841	4.748595	55.08199	60.53	60.5
15	5.8472	3.582276	59.08387	55.72	55.7
20	6.9055	2.937762	62.98051	50.92	50.9
25	7.9426	2.522288	66.59672	46.33	46.3
30	8.9563	2.223121	69.75829	41.75	41.7
35	9.9489	1.997652	72.63421	37.26	37.2
40	10.9231	1.824198	75.4214	32.97	32.9
45	11.8801	1.682129	77.93779	28.65	28.6
50	12.8226	1.568292	80.48889	24.54	24.5
55	13.7521	1.476093	83.12447	20.65	20.6
60	14.6704	1.404006	86.07383	17.13	17.1
65	15.578	1.345069	89.18236	13.84	13.8
70	16.4763	1.297951	92.75328	11.00	10.9
75	17.366	1.266	96.583	8.43	8.4
80	18.2475	1.2403	100.815	6.23	6.2
85	19.1221	1.2256	105.745	4.64	4.6

Table 3.5. Estimated α, Γ(α+1, *t*), Γ(α+2, *t*), estimated and SRS life expectancies for different ages, India (male, Total, 2012-16):

		2012-16			
Age (t)	**α**	**Γ(α+1, *t*)**	**Γ(α+2, *t*)**	**Est. L.E**	**SRS L.E**
0	4.112	28.44908	145.4317	67.44	67.4
1	3.701	15.35928	72.57186	69.19	69.1
5	3.8416	7.806929	54.1158	65.56	65.5
10	4.7861	4.771857	55.35355	60.86	60.8
15	5.8489	3.599365	59.36645	56.05	56.0
20	6.9069	2.950388	63.25154	51.22	51.2
25	7.9438	2.532212	66.85895	46.61	46.6
30	8.9576	2.233101	70.07158	42.07	42.0
35	9.9499	2.004842	72.89574	37.51	37.5
40	10.9237	1.828275	75.59004	33.13	33.1
45	11.8807	1.686005	78.11739	28.82	28.8
50	12.8233	1.572621	80.71109	24.75	24.7
55	13.7527	1.479665	83.32578	20.83	20.8
60	14.6711	1.408041	86.32233	17.35	17.3
65	15.5786	1.348746	89.41751	14.05	14.0
70	16.4769	1.300158	92.91421	11.14	11.0
75	17.366	1.266	96.583	8.43	8.4
80	18.2472	1.2386	100.682	6.12	6.1
85	19.1218	1.224	105.603	4.52	4.5

Table 3.6. Estimated α, Γ(α+1, *t*), Γ(α+2, *t*), estimated and SRS life expectancies for different ages, India (male, Total, 2013-17):

		2013-17			
Age (t)	**α**	**Γ(α+1, *t*)**	**Γ(α+2, *t*)**	**Est. L.E**	**SRS L.E**
0	4.116	28.62384	146.4396	67.90	67.8
1	3.704	15.42616	72.93254	69.53	69.5
5	3.8441	7.8443	54.38213	65.88	65.8
10	4.7876	4.789378	55.55811	61.11	61.1
15	5.8498	3.608445	59.5166	56.22	56.2
20	6.9084	2.963975	63.54322	51.54	51.5
25	7.9448	2.540511	67.07826	46.84	46.8
30	8.9584	2.239264	70.26508	42.26	42.2

35	9.9507	2.010613	73.10564	37.72	37.7
40	10.9242	1.831681	75.73086	33.27	33.2
45	11.8811	1.688593	78.23735	28.93	28.9
50	12.8236	1.574479	80.80651	24.83	24.8
55	13.7531	1.48205	83.46027	20.95	20.9
60	14.6714	1.409783	86.42877	17.45	17.4
65	15.5786	1.348746	89.41751	14.05	14.0
70	16.4769	1.300158	92.91421	11.14	11.0
75	17.366	1.266	96.583	8.43	8.4
80	18.247	1.2375	100.593	6.05	6.0
85	19.1213	1.2212	105.368	4.33	4.3

Table 4. Estimated α, Γ(α+1, *t*), Γ(α+2, *t*), estimated life expectancies for different ages, India (male, Total, 2014-18):

		2014-18		
Age (t)	**α**	**Γ(α+1, *t*)**	**Γ(α+2, *t*)**	**Est. L.E**
0	4.121446	28.86367	147.8237	68.52
1	3.708106	15.51822	73.42931	69.99
5	3.84664	7.882455	54.65409	66.21
10	4.788663	4.801834	55.70352	61.29
15	5.851305	3.62368	59.76853	56.51
20	6.910109	2.979532	63.87719	51.90
25	7.945599	2.547162	67.254	47.02
30	8.964739	2.288706	71.81733	43.84
35	9.951768	2.018342	73.38679	38.00
40	10.9253	1.83916	76.04018	33.56
45	11.88144	1.69081	78.34007	29.03
50	12.82443	1.579621	81.07046	25.08
55	13.75441	1.489924	83.90353	21.35
60	14.67127	1.409063	86.38362	17.41
65	15.57962	1.354714	89.79487	14.38
70	16.47669	1.299393	92.85839	11.09
75	17.35687	1.2171	92.847	5.22
80	18.24826	1.24465	101.17	6.54
85	19.12603	1.2471	107.599	6.21

Table 5. Estimated α, Γ(α+1, *t*), Γ(α+2, *t*), estimated life expectancies for different ages, India (male, Total, 2015-19):

2015-19				
Age (t)	**α**	$\boldsymbol{\Gamma(\alpha+1,t)}$	$\boldsymbol{\Gamma(\alpha+2,t)}$	**Est. L.E**
0	4.125932	29.06286339	148.97426151	69.03
1	3.711592	15.59685832	73.8539123	70.75
5	3.84908	7.91928391	54.91663979	66.52
10	4.790494	4.82336553	55.95489891	61.59
15	5.852822	3.63910191	60.02354942	56.80
20	6.911395	2.99129263	64.12964720	52.18
25	7.946676	2.55615455	67.49162554	47.27
30	8.965656	2.29594816	72.04470069	44.06
35	9.952591	2.02431932	73.60419136	38.21
40	10.925978	1.84384045	76.23375403	33.75
45	11.881999	1.69442558	78.50764371	29.18
50	12.824911	1.58262873	81.22482064	25.21
55	13.754863	1.49262127	84.05553181	21.48
60	14.671708	1.411570429	86.53838829	17.54
65	15.579942	1.35620905	89.91646066	14.48
70	16.476829	1.30295315	92.89266696	11.12
75	17.35662	1.21558002	92.73473915	5.12
80	18.24737	1.23956984	100.7570837	6.18
85	19.12529	1.24315685	107.2591475	5.92

Table 6. Estimated α, $\Gamma(\alpha+1,t)$, $\Gamma(\alpha+2,t)$, estimated life expectancies for different ages, India (male, Total, 2016-20)

		2016-20		
Age (t)	**α**	$\boldsymbol{\Gamma(\alpha+1,t)}$	$\boldsymbol{\Gamma(\alpha+2,t)}$	**Est. L.E**
0	4.130418	29.26356202	150.13430537	69.55
1	3.715078	15.67593488	74.281135153	70.78
5	3.85152	7.956287723	55.180471540	66.84
10	4.792325	4.844993584	56.207411900	61.89
15	5.854339	3.654589396	60.279657762	57.10
20	6.912681	3.003099397	64.383107223	52.45
25	7.947753	2.565178780	67.730087805	47.52
30	8.966573	2.303213210	72.272790754	44.29
35	9.953414	2.030313864	73.822233040	38.42
40	10.926661	1.848533002	76.427820538	33.93
45	11.882556	1.698049338	78.675577612	29.34
50	12.825394	1.585641708	81.379479580	25.36
55	13.755312	1.495325035	84.207811167	21.62
60	14.672142	1.414094028	86.693117798	17.68
65	15.580268	1.358064590	90.039493505	14.59
70	16.476966	1.303714992	92.946985589	11.17
75	17.35637	1.214263536	92.634300161	5.03
80	18.24648	1.234727567	100.36346291	5.84
85	19.12455	1.239063009	106.90591476	5.62

Table 7. Estimated α, $\Gamma(\alpha+1,t)$, $\Gamma(\alpha+2,t)$, estimated life expectancies for different ages, India (male, Total, 2017-21)

		2017-21		
Age (t)	**α**	$\boldsymbol{\Gamma(\alpha+1,t)}$	$\boldsymbol{\Gamma(\alpha+2,t)}$	Est**. L.E**
0	4.134904	29.46577418	151.30392174	70.07
1	3.718564	15.75545450	74.710999855	71.18
5	3.85396	7.993466767	55.445590125	67.16
10	4.794156	4.866718847	56.461067624	62.20
15	5.855856	3.670142850	60.536859930	57.39
20	6.913967	3.014952784	64.637569453	52.73
25	7.94883	2.574234875	67.969392816	47.77
30	8.96749	2.310501246	72.501603053	44.53
35	9.954237	2.036326158	74.040920700	38.64
40	10.927344	1.853237495	76.622381105	34.12
45	11.883113	1.701680845	78.843870747	29.50
50	12.825877	1.588660413	81.534433008	25.50
55	13.755761	1.498033690	84.360366406	21.76
60	14.672576	1.416622139	86.848123961	17.82
65	15.580594	1.361897517	90.293638666	14.81
70	16.477103	1.304477277	93.001335981	11.22
75	17.35612	1.212948478	92.533969948	4.94
80	18.24559	1.229904208	99.971379834	5.51
85	19.12381	1.234982643	106.55384530	5.32

Table 8. Estimated α, $\Gamma(\alpha+1,t)$, $\Gamma(\alpha+2,t)$, estimated life expectancies for different ages, India (male, Total, 2018-22):

		2018-22		
Age (t)	**α**	$\boldsymbol{\Gamma(\alpha+1,t)}$	$\boldsymbol{\Gamma(\alpha+2,t)}$	**Est. L.E**
0	4.13939	29.66951196	152.48319312	70.60
1	3.72205	15.83541982	75.143523615	71.58
5	3.8564	8.030821885	55.712001929	67.48
10	4.795987	4.888541765	56.715871276	62.51
15	5.857373	3.685762554	60.795160601	57.69
20	6.915253	3.026852976	64.893037858	53.01
25	7.949907	2.583322949	68.209543560	48.02
30	8.968407	2.317812346	72.731139875	44.76
35	9.95506	2.042356258	74.260256259	38.85
40	10.928027	1.857953961	76.817436998	34.30
45	11.88367	1.705320119	79.012523895	29.66
50	12.82636	1.591684866	81.689681492	25.64
55	13.75621	1.500747252	84.513198032	21.90
60	14.67301	1.419154770	87.003407280	17.96
65	15.58092	1.361783285	90.286064475	14.80
70	16.47724	1.305240008	93.055718154	11.26
75	17.35587	1.211634844	92.433748403	4.86
80	18.2447	1.225099692	99.580828494	5.18
85	19.12307	1.230915715	106.20293532	5.03

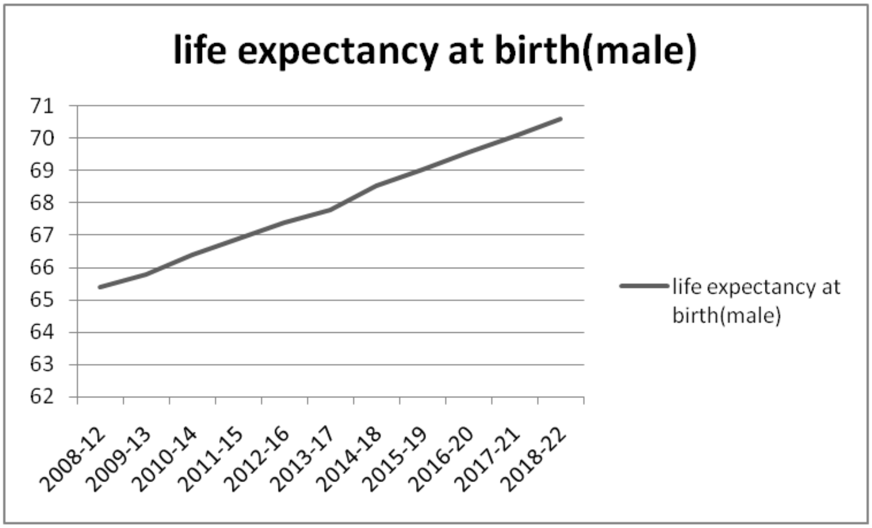

Figure 1. Life expectancy at birth (male), 2008-2022

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